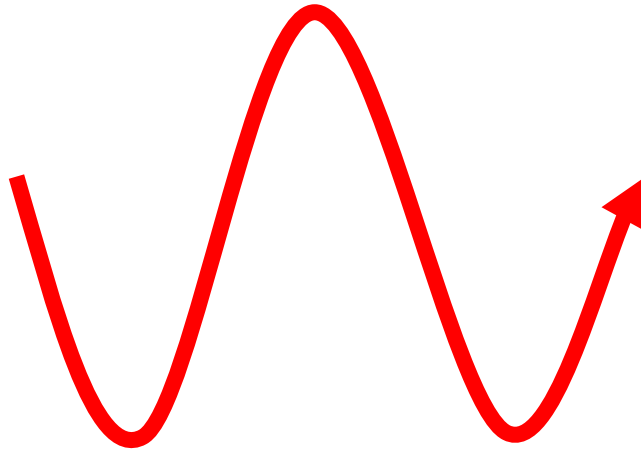


TOPOLOGICAL PHYSICS

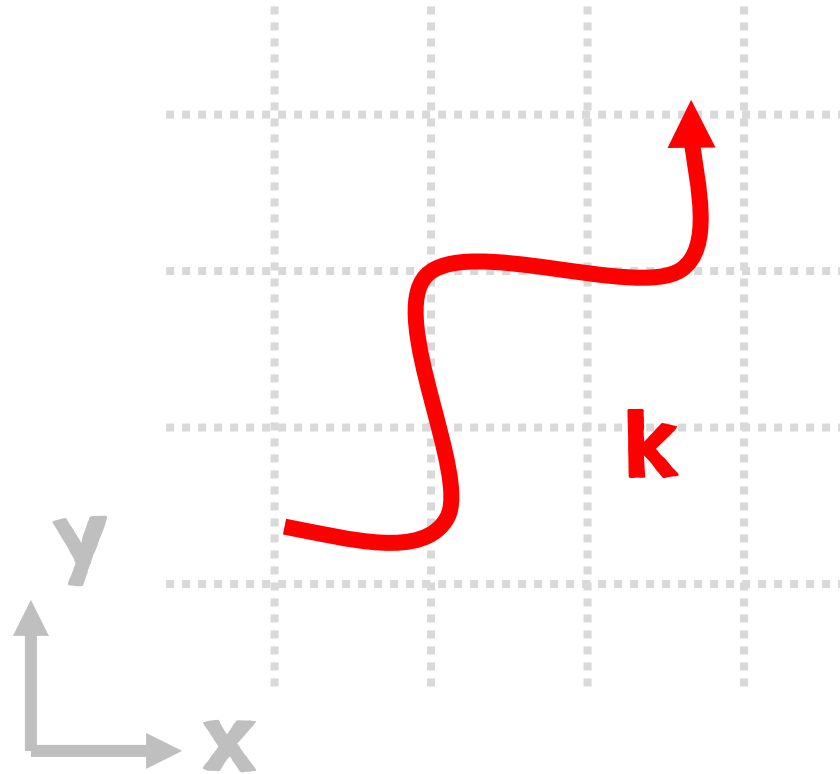
for
babies



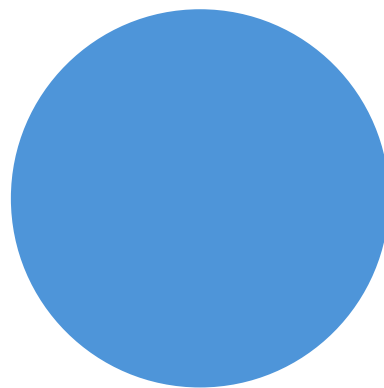
Junda Wang



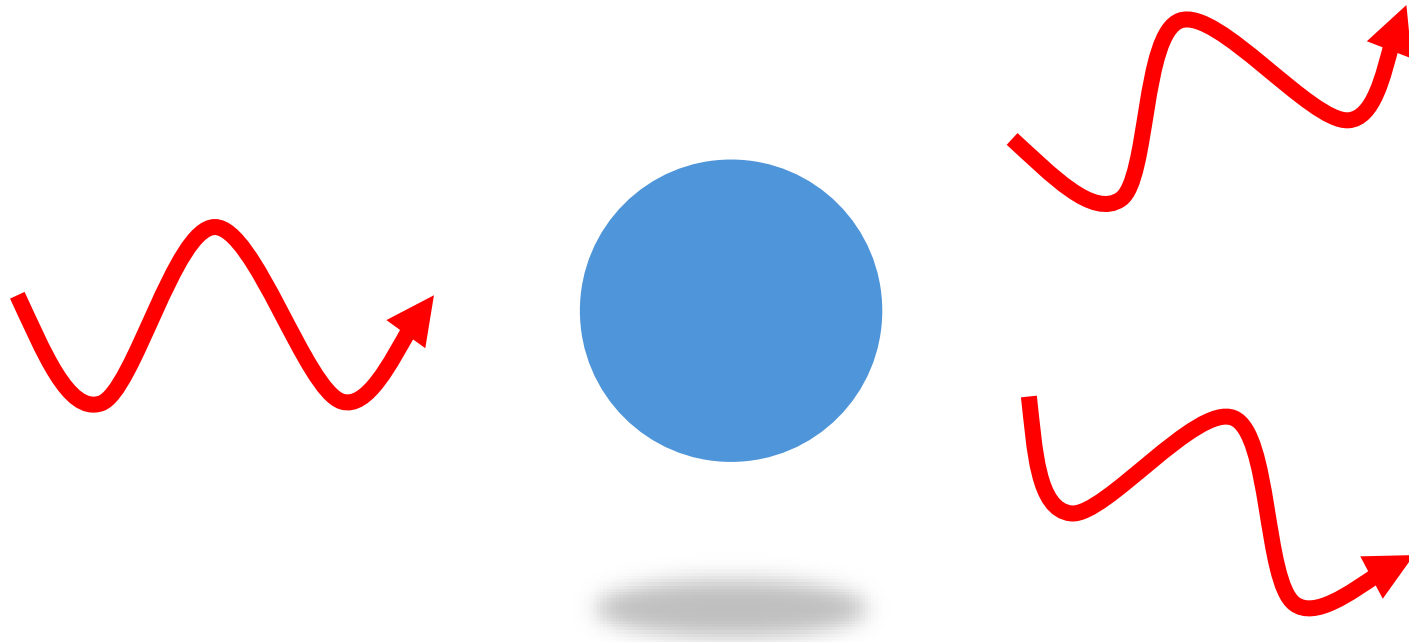
This is a wave.



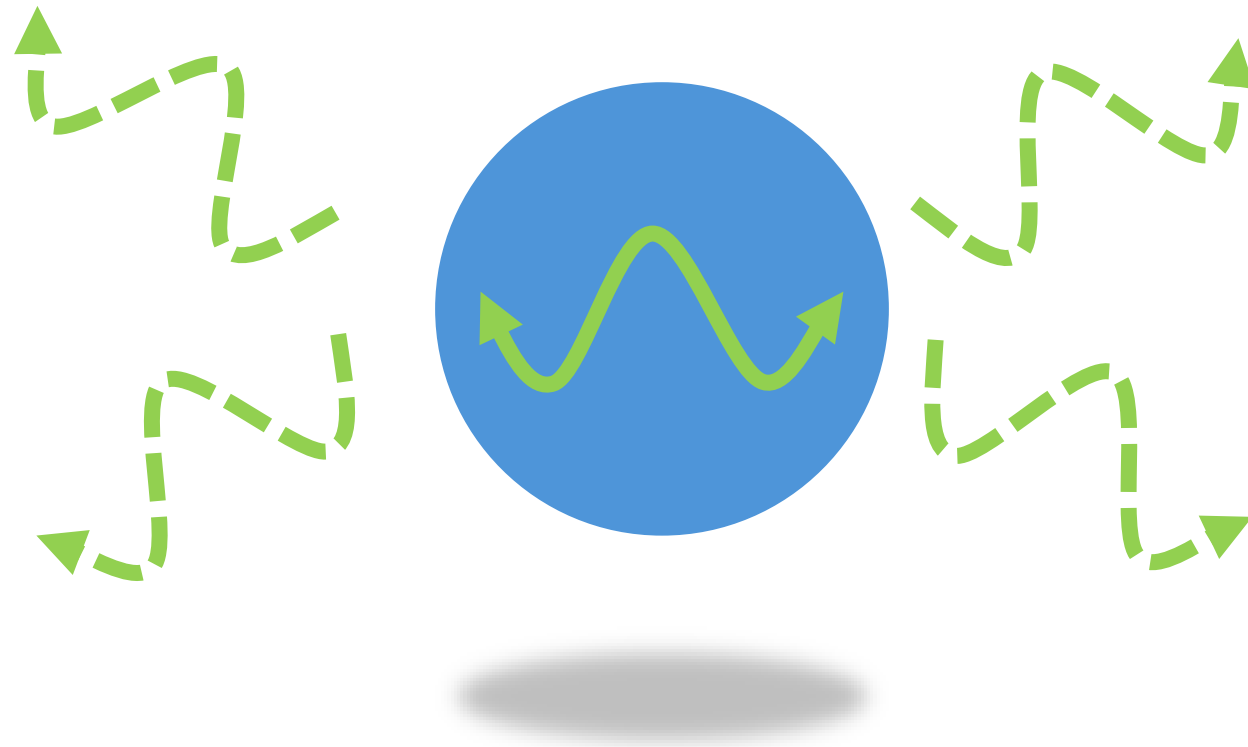
**Wave travels across space.
It has a momentum.**



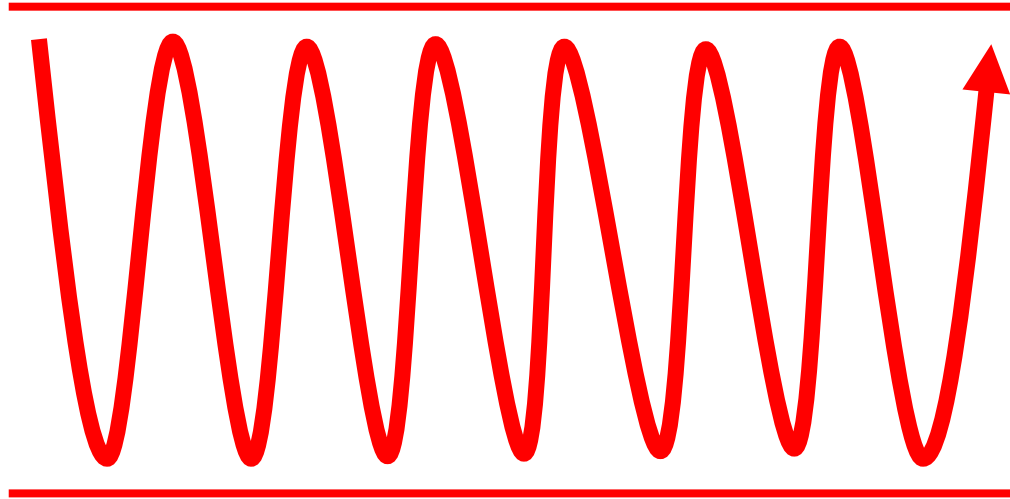
This is a ball.



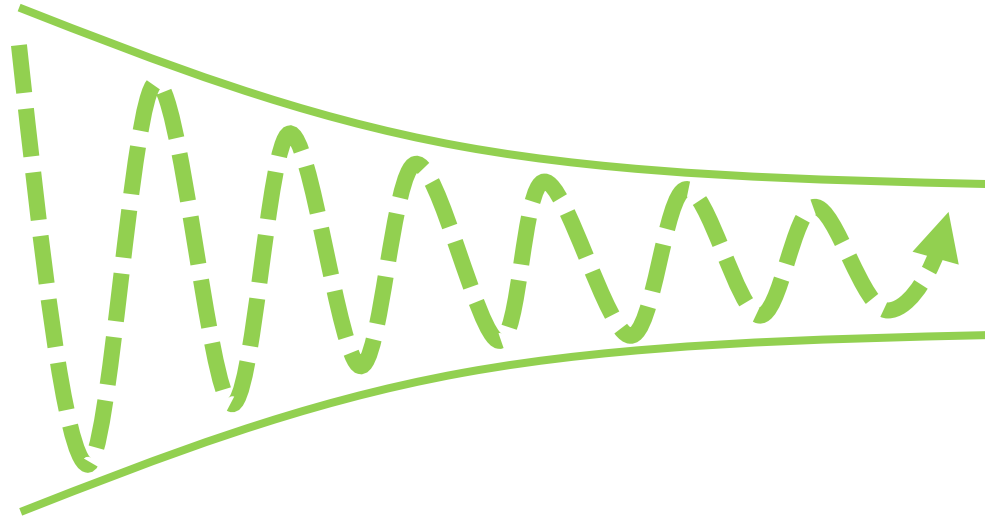
**Wave hits the ball.
It will be **scattered** but still **travel**.**



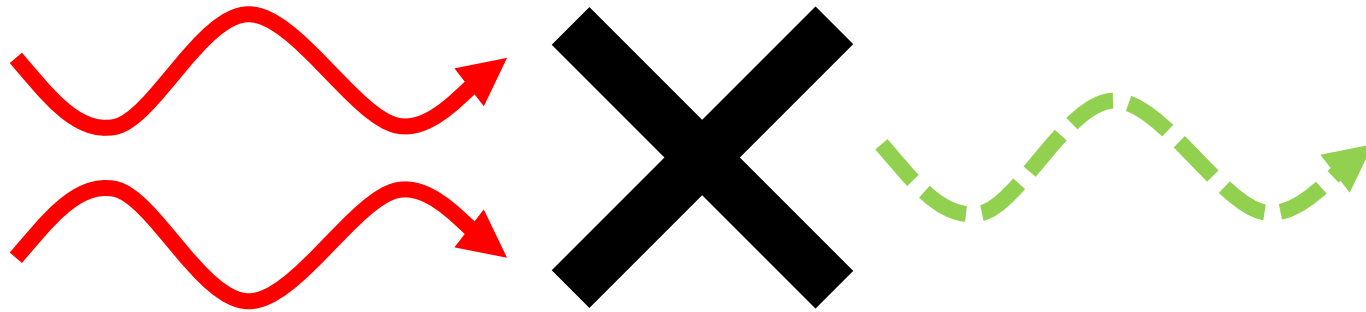
**If wave stays in the ball,
it will be trapped.**



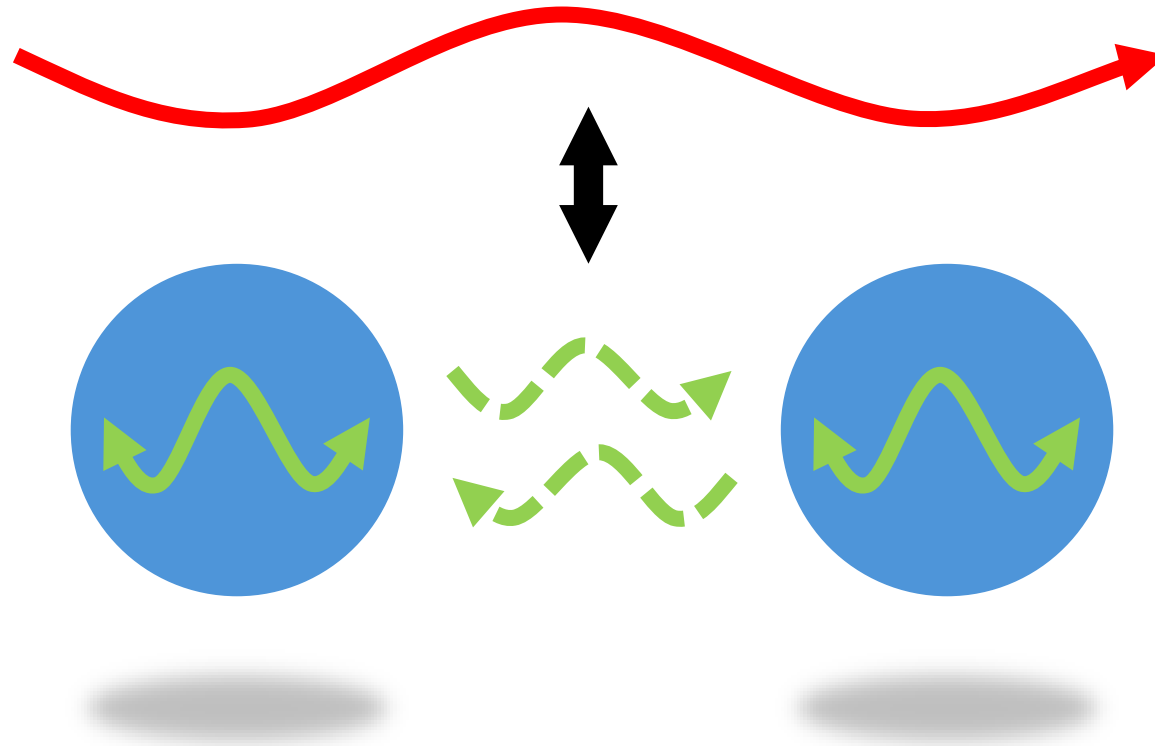
Traveling wave runs with a **constant** amplitude.
We say it has a **real** wave momentum, k .



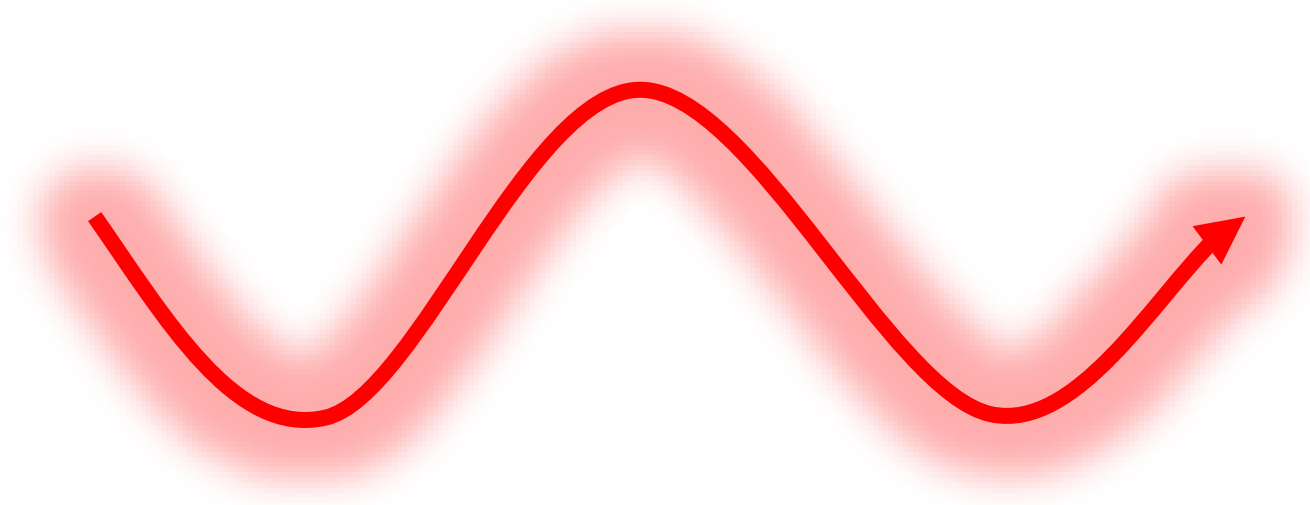
Trapped wave runs with a **decreasing** amplitude.
We say it has an **imaginary** wave momentum, $\sqrt{-1} \cdot k$.



When two **traveling** waves meet,
they may interfere and get **trapped**.



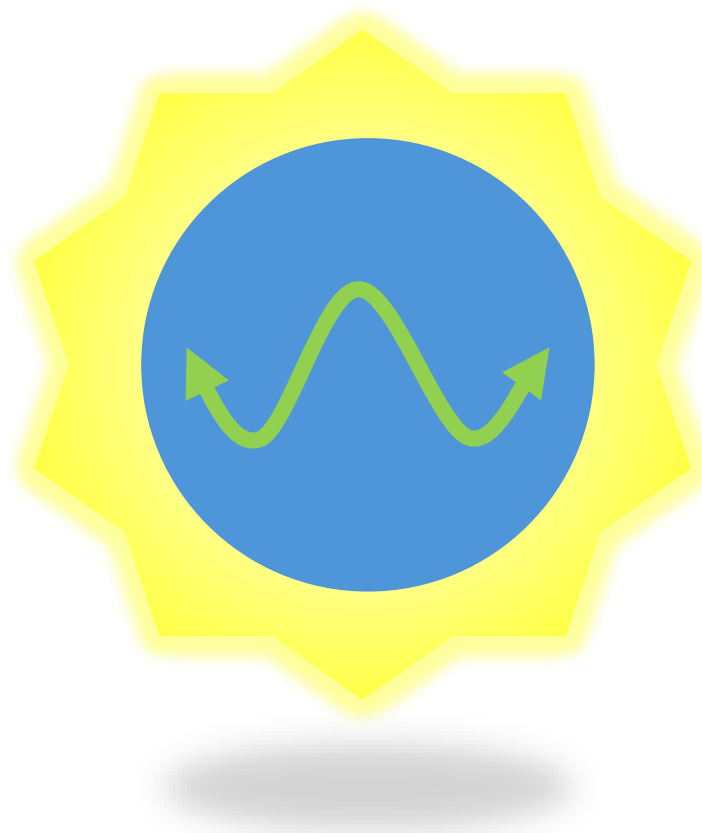
When two **trapped** waves meet,
they may hop and begin to **travel**.



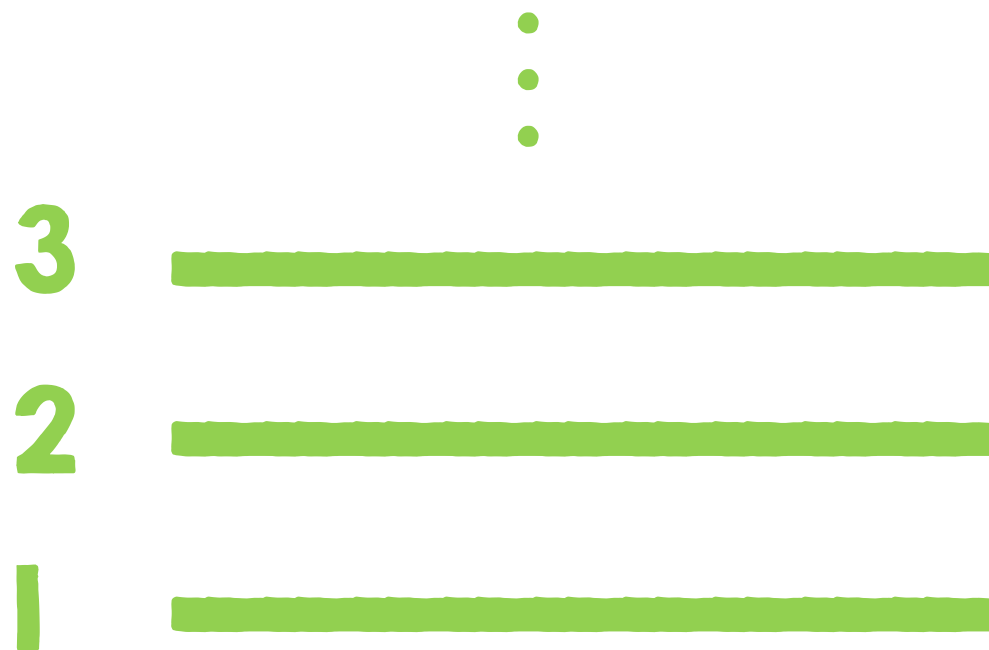
Traveling wave has energy.



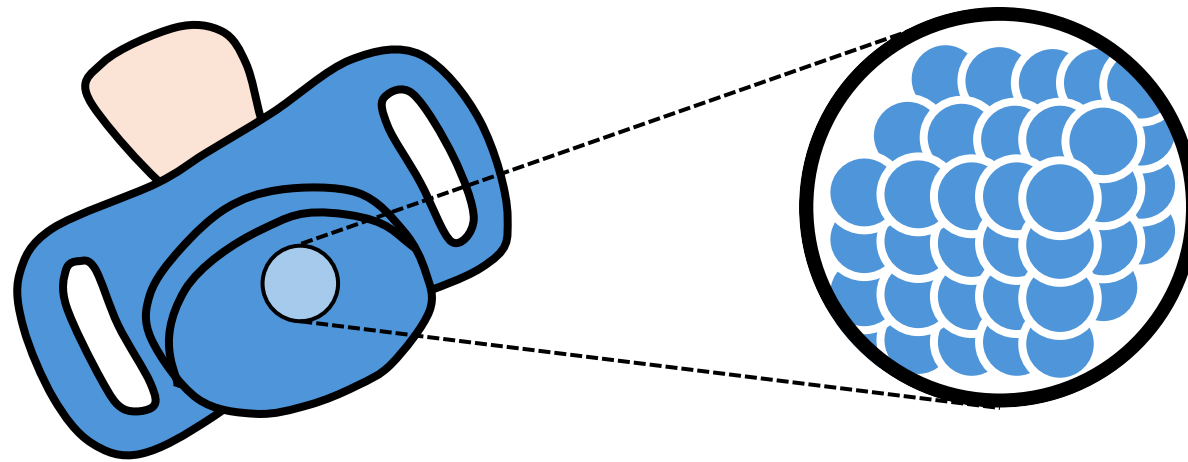
It's continuous like the rainbow.



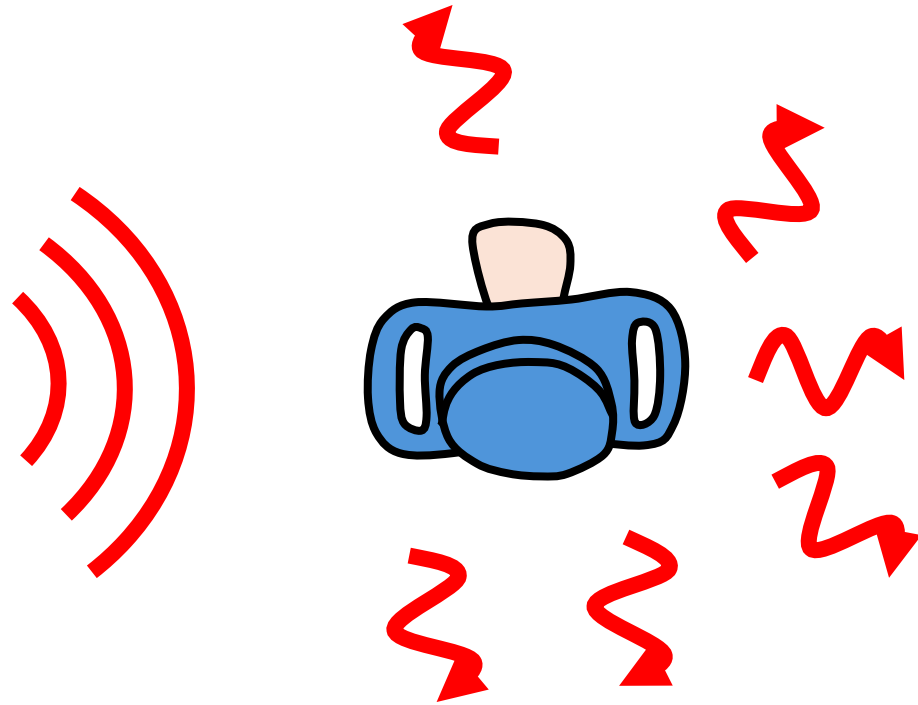
Trapped wave also has energy.



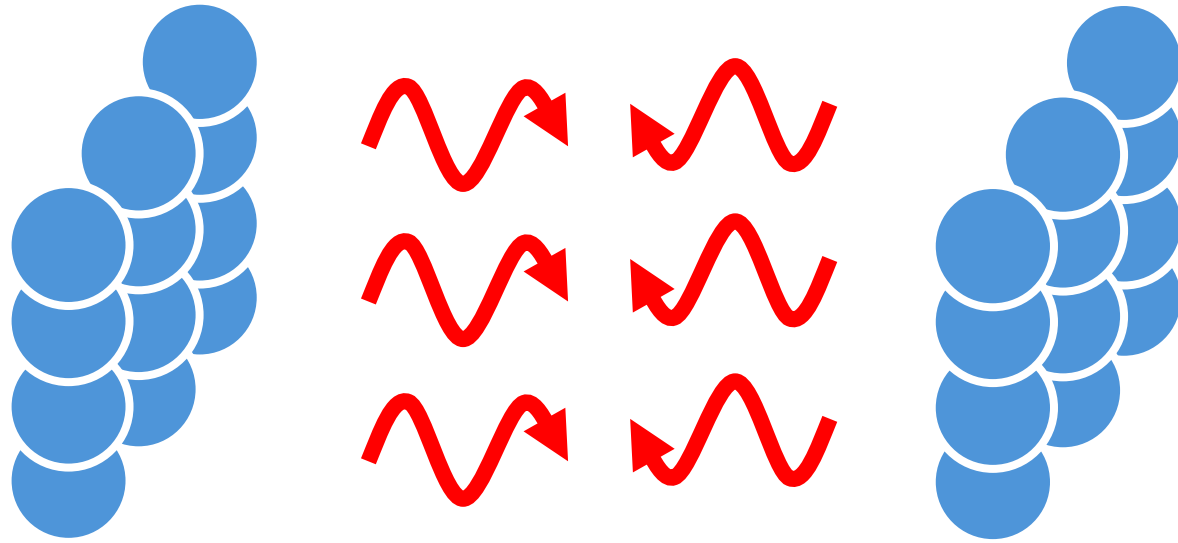
It's discrete as 1, 2, 3.



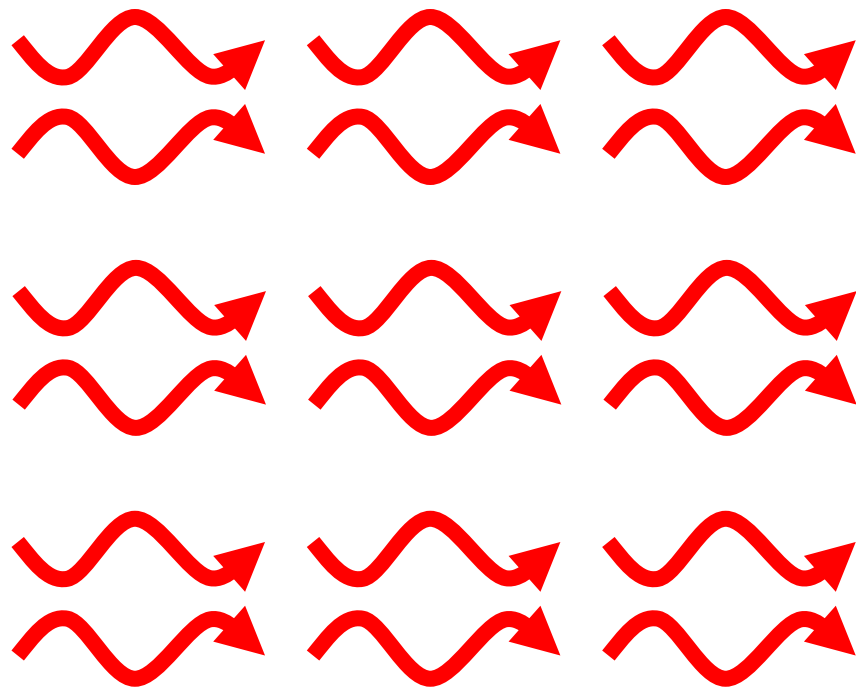
In materials, there are so many balls.



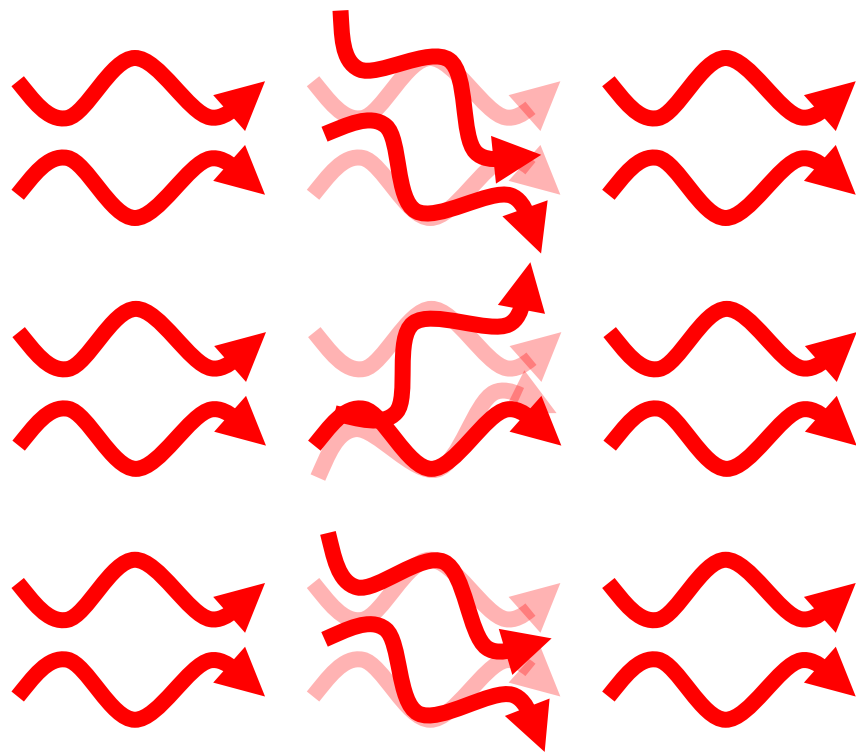
When wave wants to travel in the materials, it is shredded into so many baby waves by balls.



**Balls distribute in an organized way.
Baby waves also meet each other in an organized way.**



When the energies of baby waves satisfy certain condition, they always meet in an unpleasant way and **trapped.**



**The number of baby waves is huge.
Small imperfection won't shake the result.**



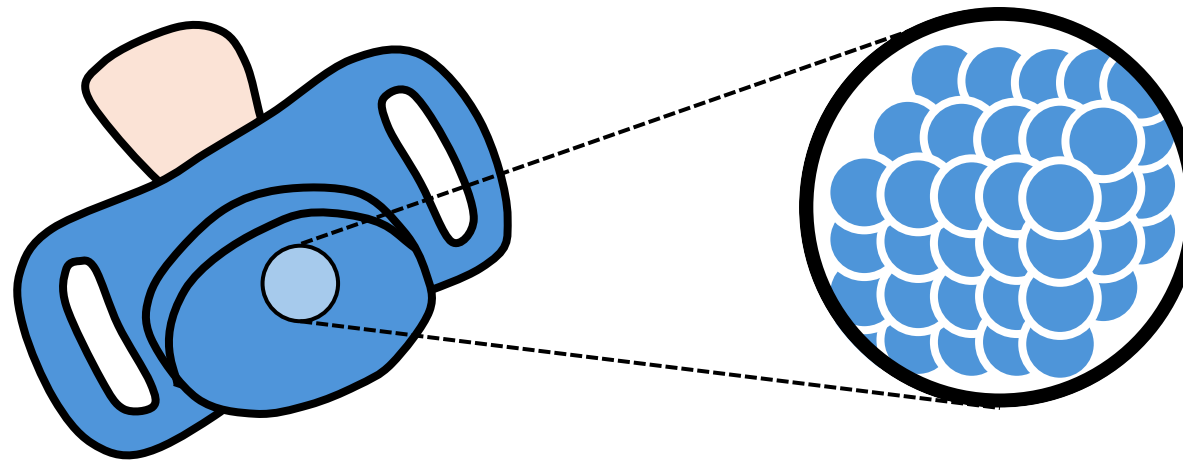
**Now the continuous rainbow loses a part.
Forbidden energies form the **band gap**.**



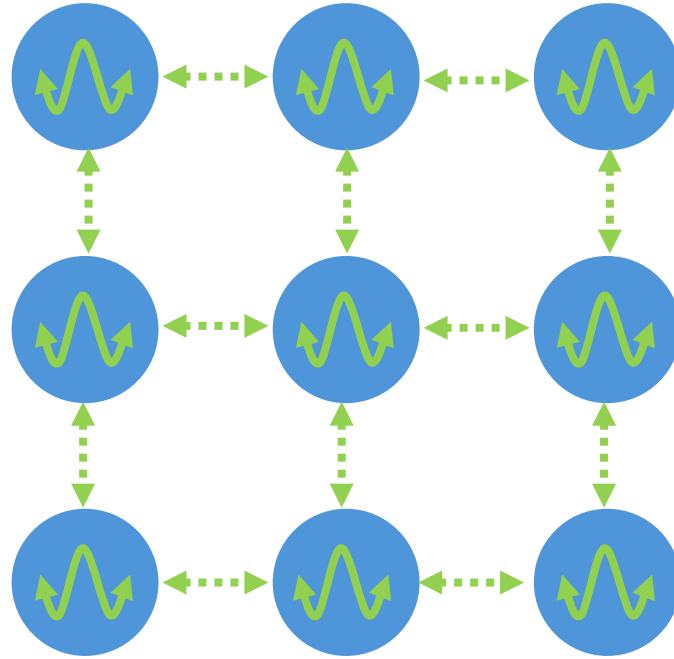
You may ask:
“What about the **trapped wave?”**

BINGO!

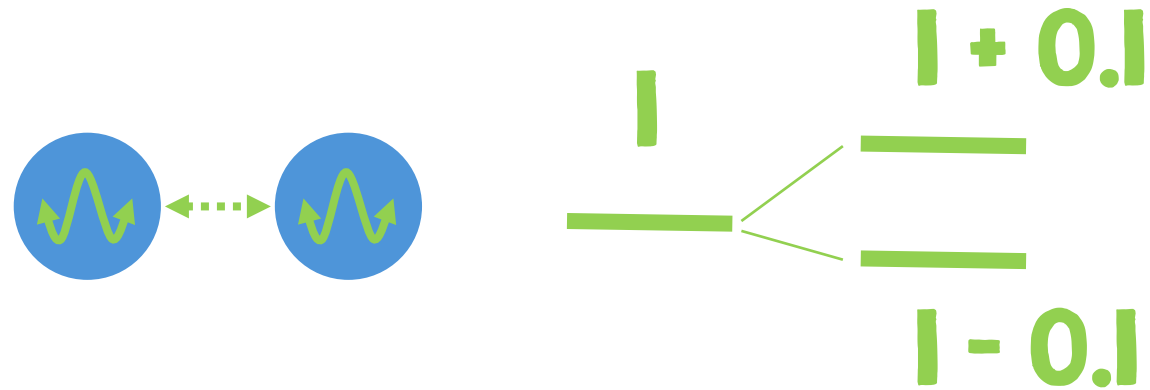
**Well, of course.
There is a different story.**



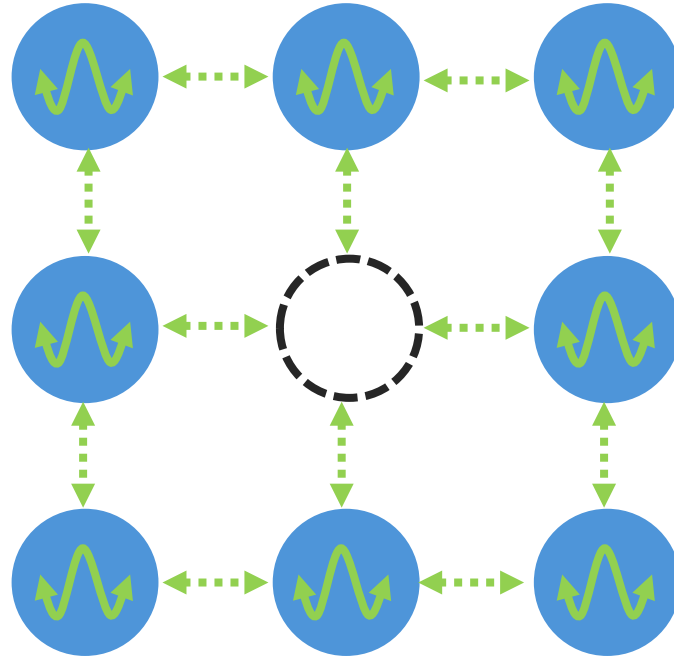
**Let's rewind a little bit.
In materials, there are so many balls.**



This time we let wave stay in those balls.
They may **travel** by hopping to their neighbors.



Hopping usually leads to a tiny energy shift.

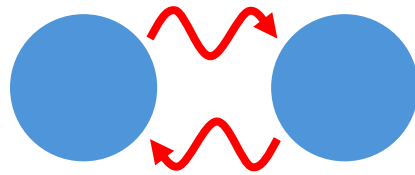


**Similarly, the number of balls is huge.
Small imperfection won't change a lot.**

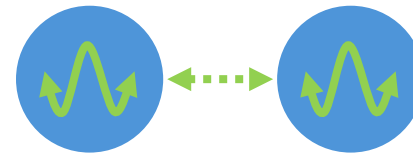


Now the large number of discrete energies forms the **band.
Areas without any energy is the **band gap**.**

NFE



TB

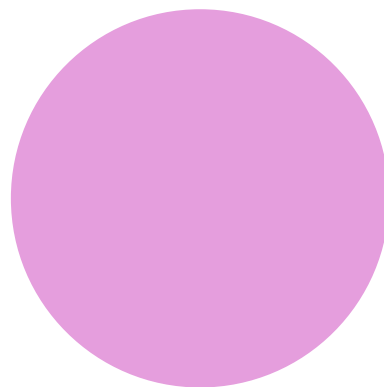


We call the first story **nearly-free-electron model**,
and the second story **tight-binding model**.

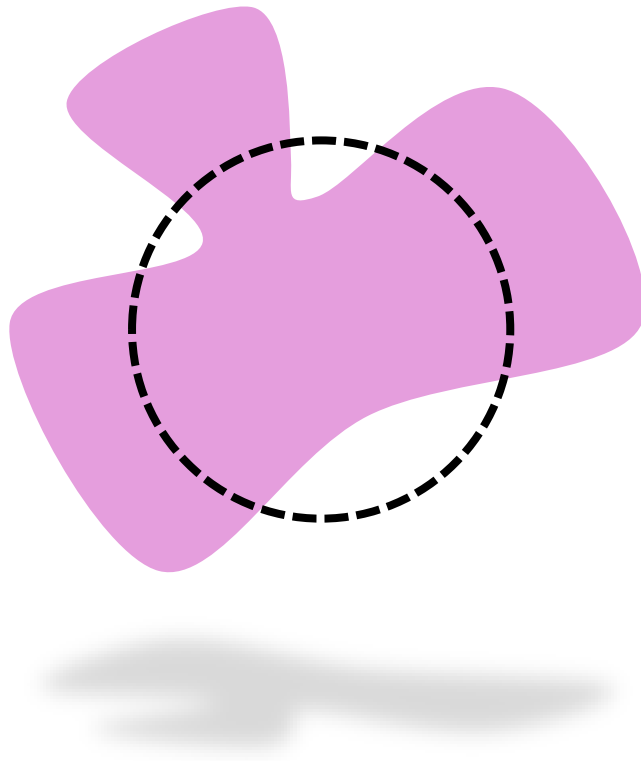


Now you know

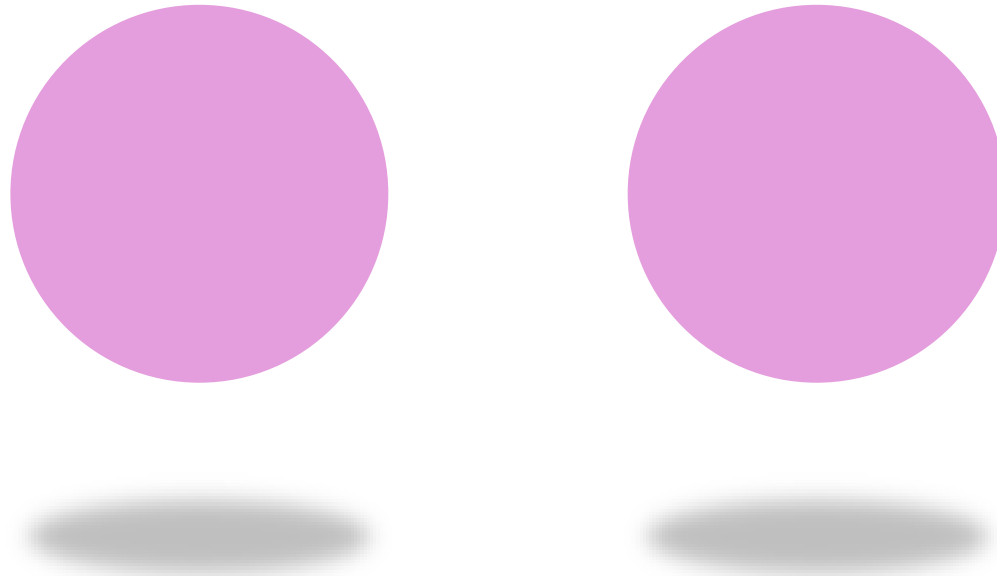
**BAND
THEORY!**



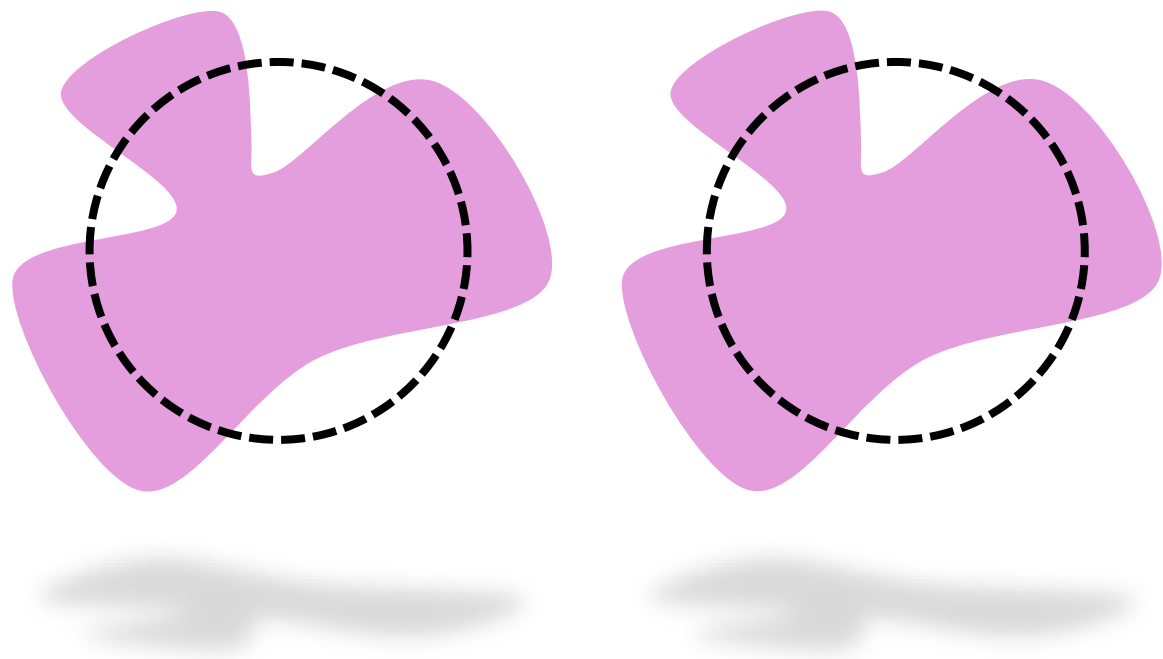
This is a ball.



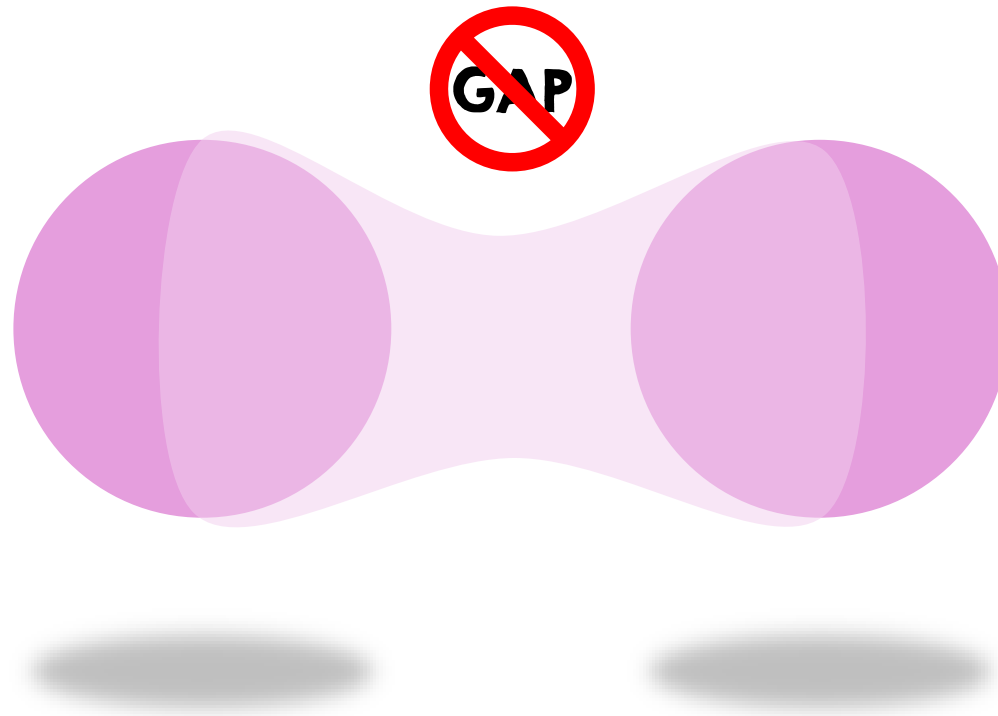
We can squash it.



These are two balls.



**We can squash them.
But they are still two.**



How can we turn them into one ball?
We glue them!

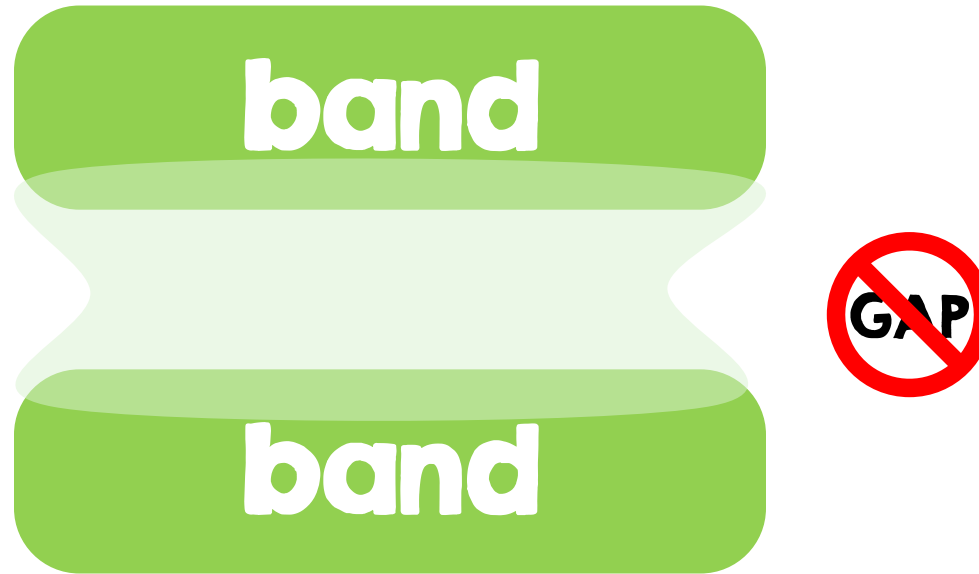
band

This is a band.

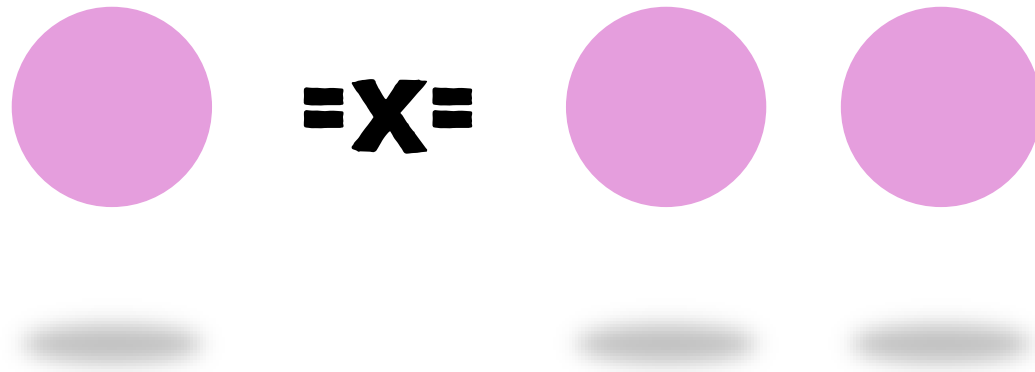
band

band

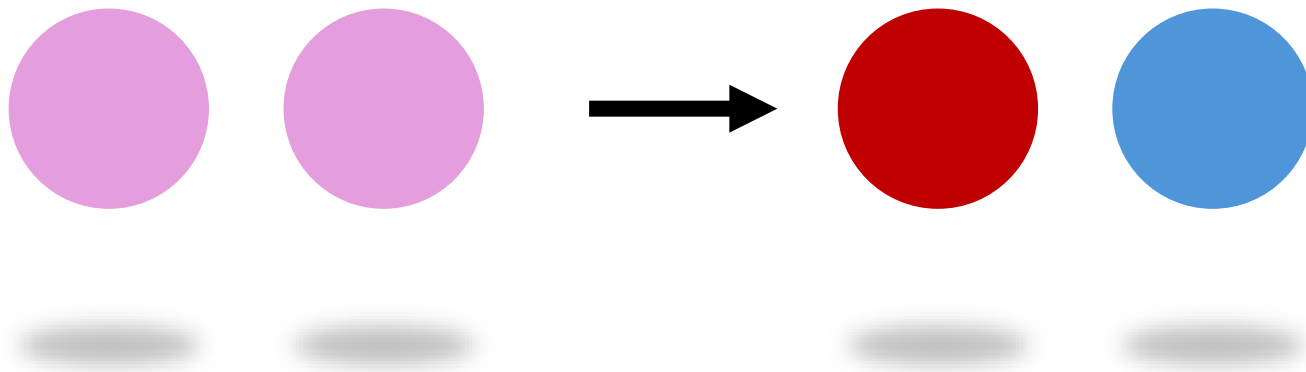
**These are two bands.
How can we turn them into one band?**



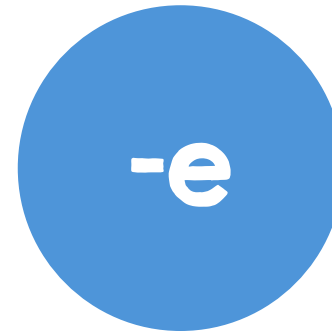
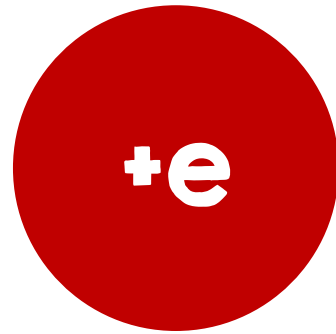
**No matter how we do it,
we close the gap.**



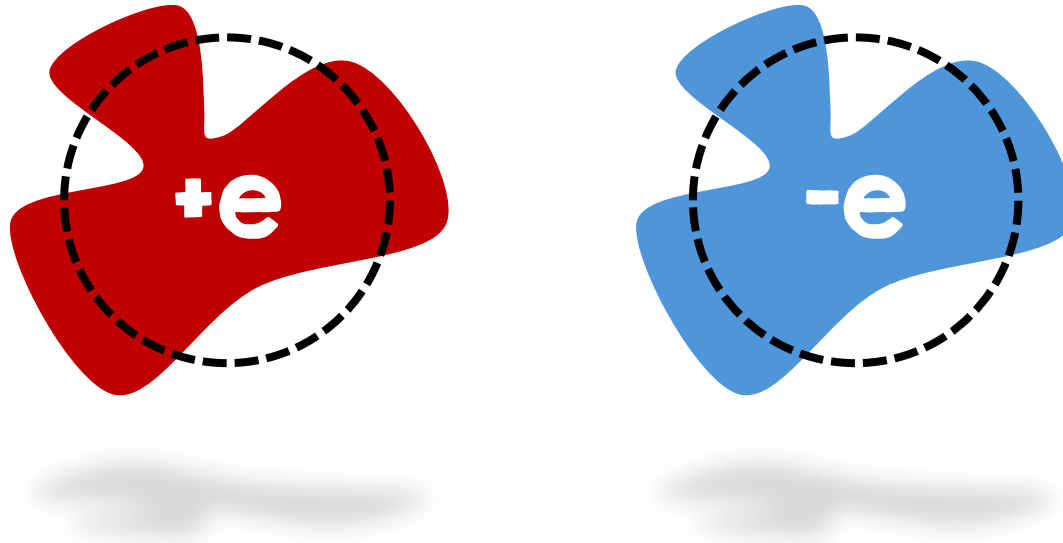
**But this is not yet the topology we want to discuss,
as the number of objects changes.**



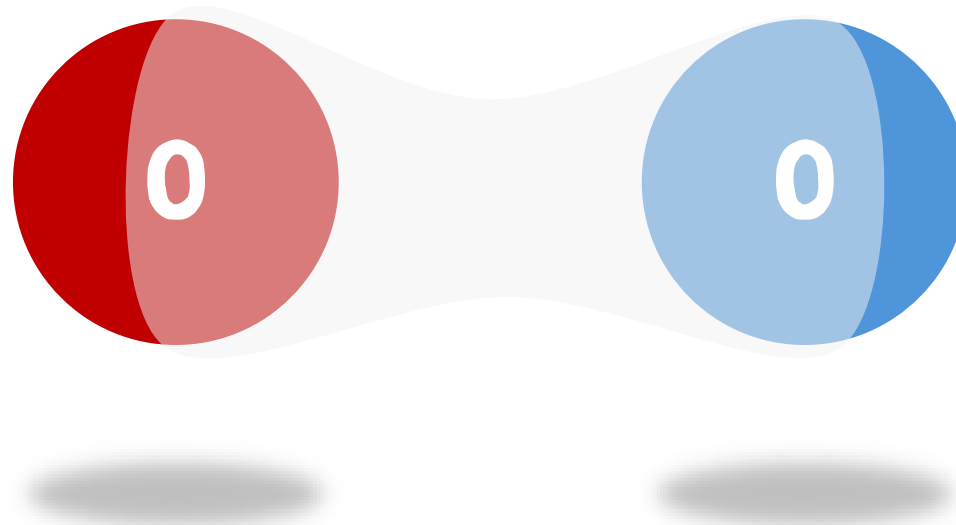
We are more interested in the situation where the number doesn't change but something else changes.



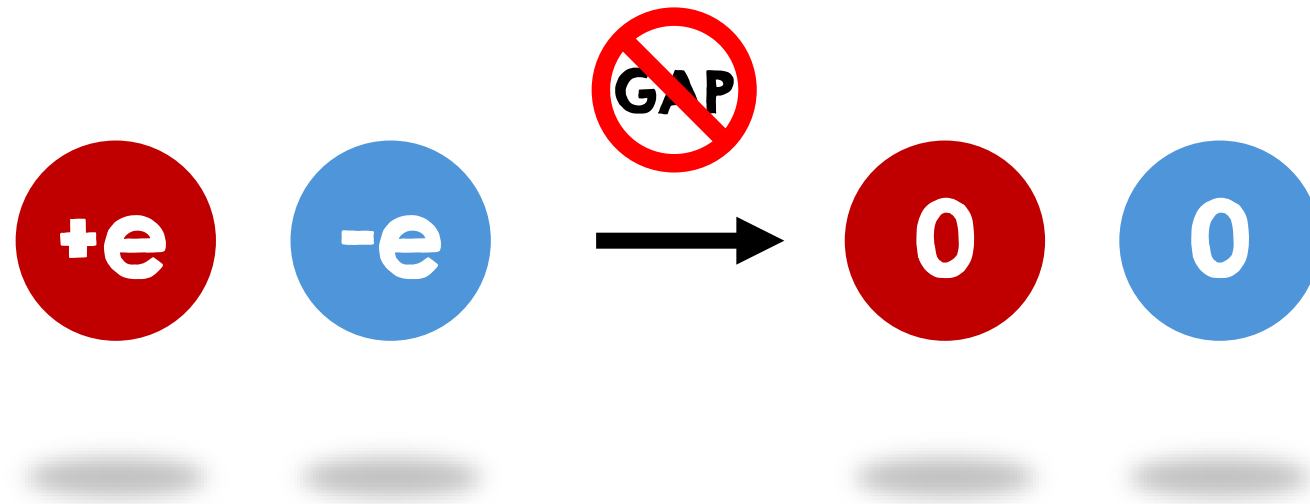
Let's think about charges.
We give the red ball $+e$ and the blue ball $-e$.



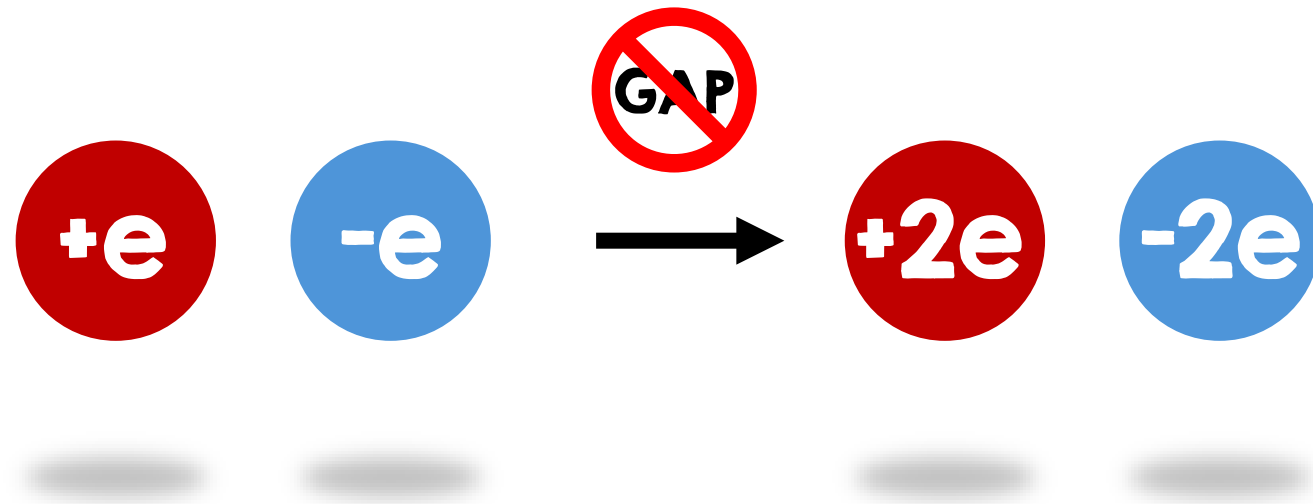
**When we squash them,
the charges won't change.**



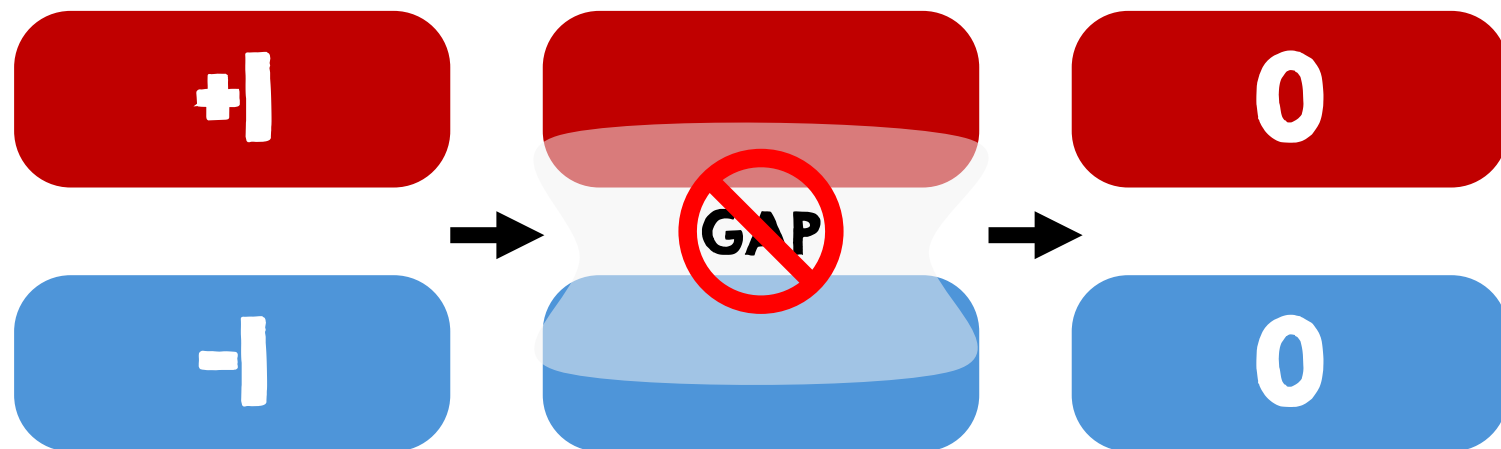
**But if we glue them,
the charges will annihilate.**



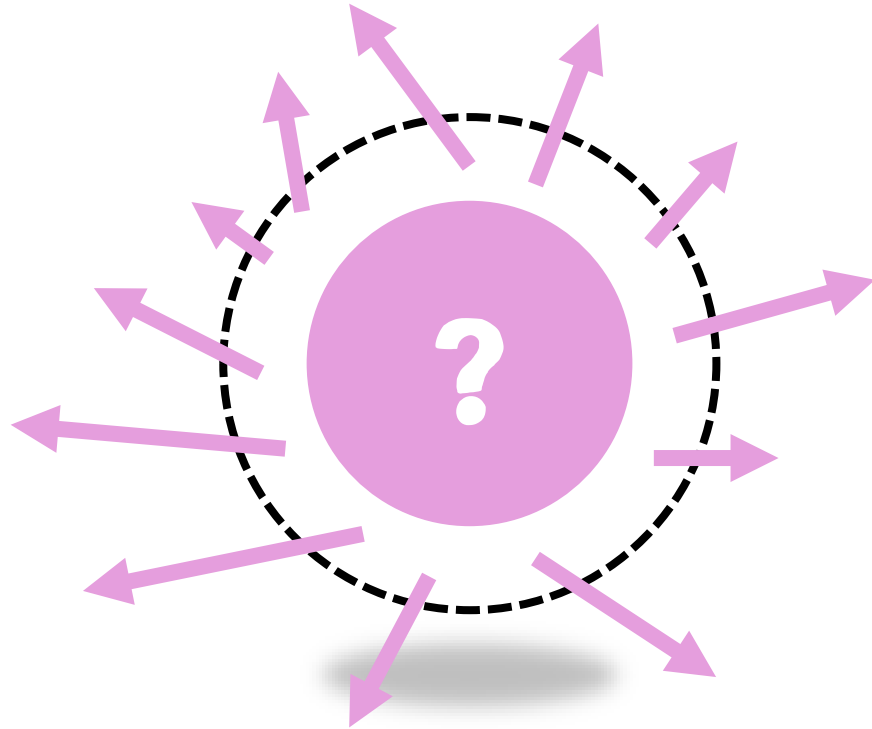
The charges only change with gap closing.
This is a “**topological**” effect.



Of course, this gap closing can bring us something else.
We call them different “**topological phases**”.



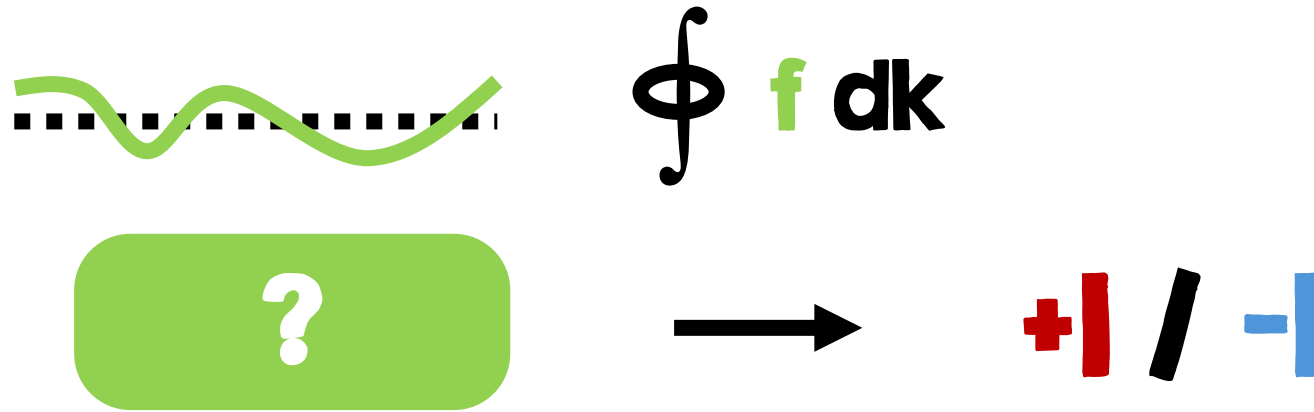
This may also work for band structures.
We can classify them based on “**topological charges**”.



$$\oint \mathbf{E} \, d\mathbf{S}$$

$$\rightarrow +e/-e$$

According to Gauss's law, we may decide the total charge number by looking at the **E field** distribution.

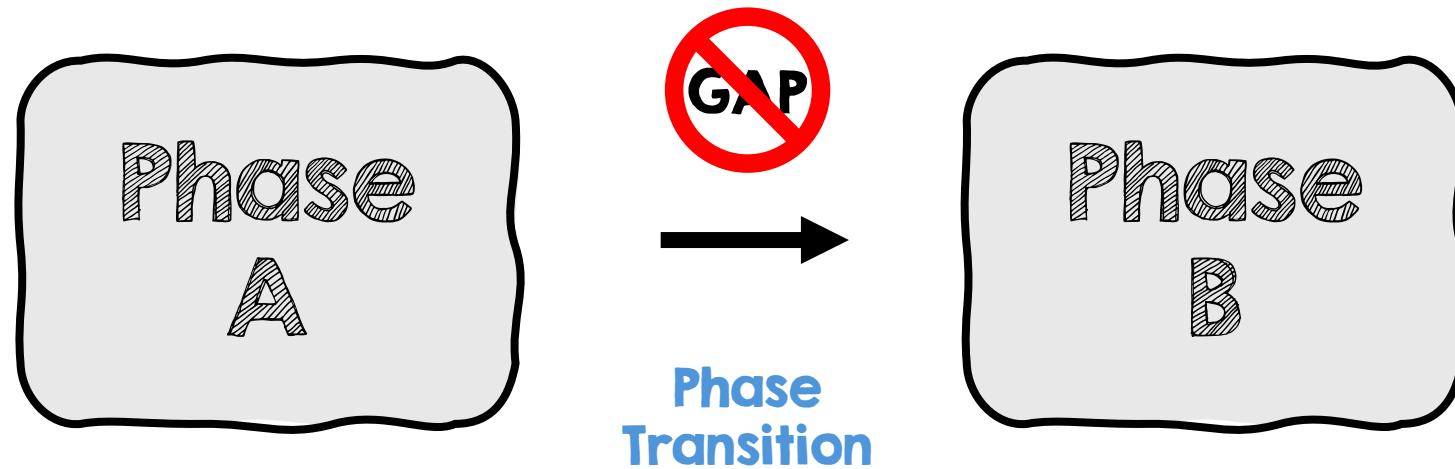


The topological charge of a band can be obtained by looking at the **wave function** distribution as well.

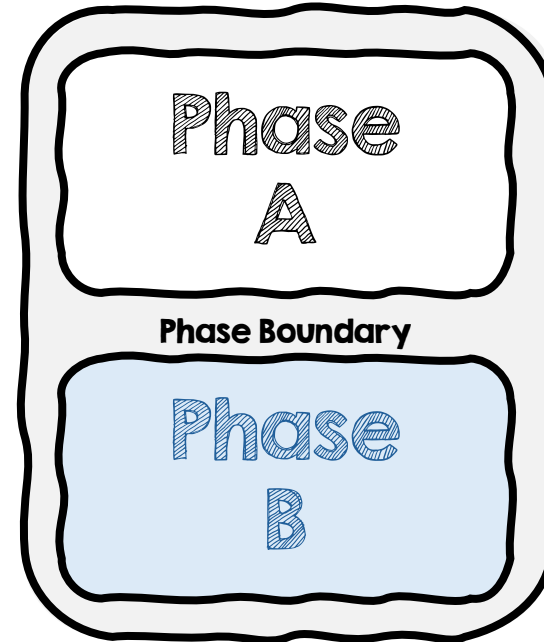
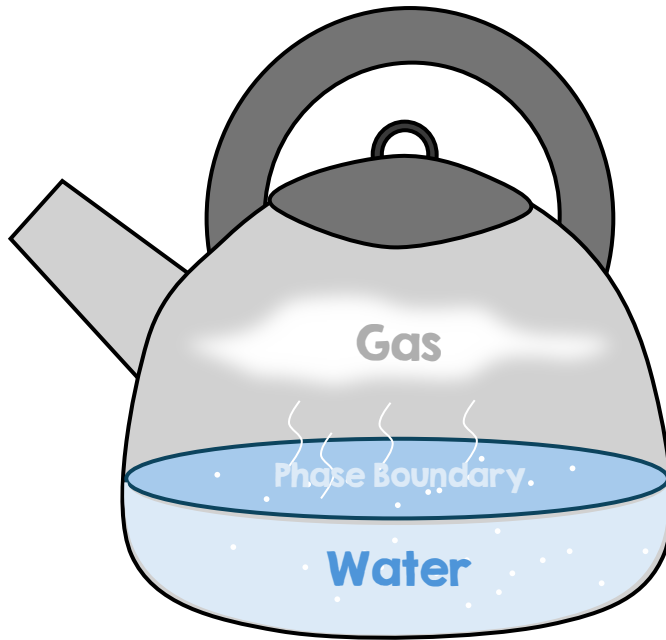
Z group

Phase A	Phase B	Phase C	Phase D	Phase E
+2	+1	0	-1	-2
-2	-1	0	+1	+2

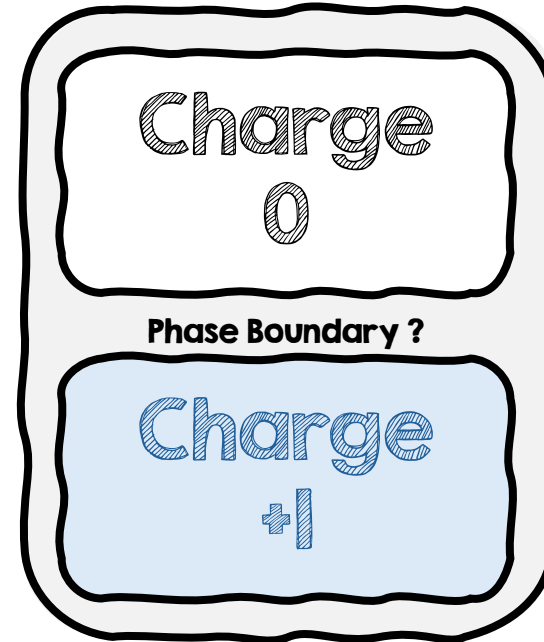
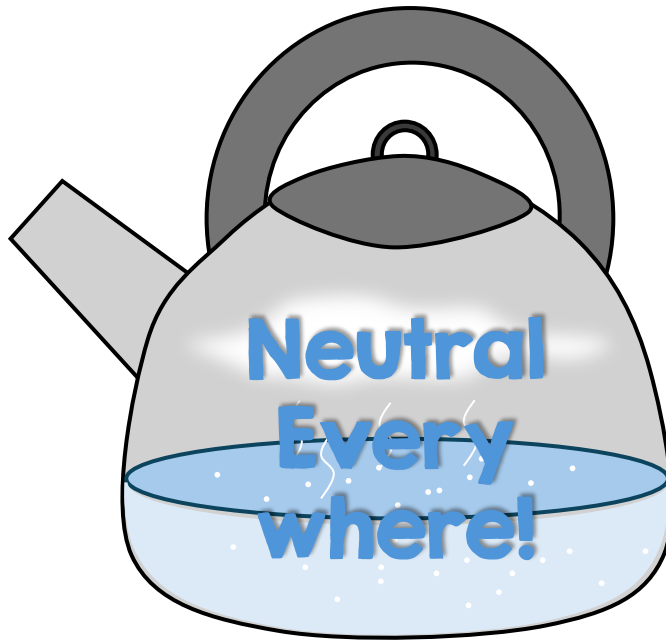
We may classify systems into distinct topological phases, even sometimes they may have identical band structures.



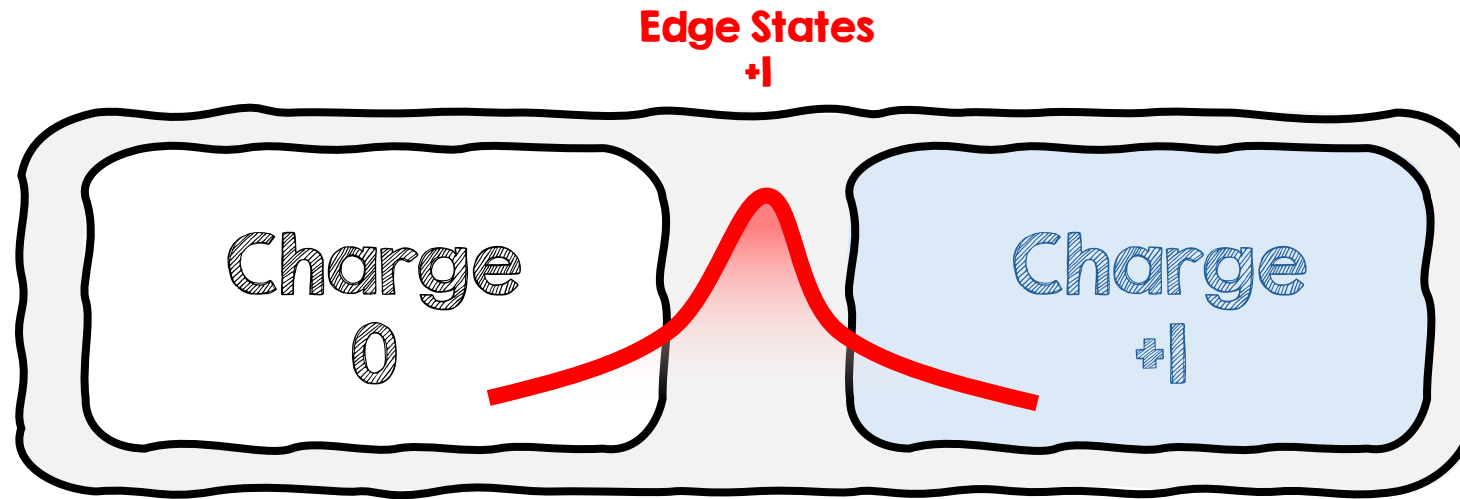
Connecting two distinct phases always needs a gap closing. We call it “**topological phase transition**”.



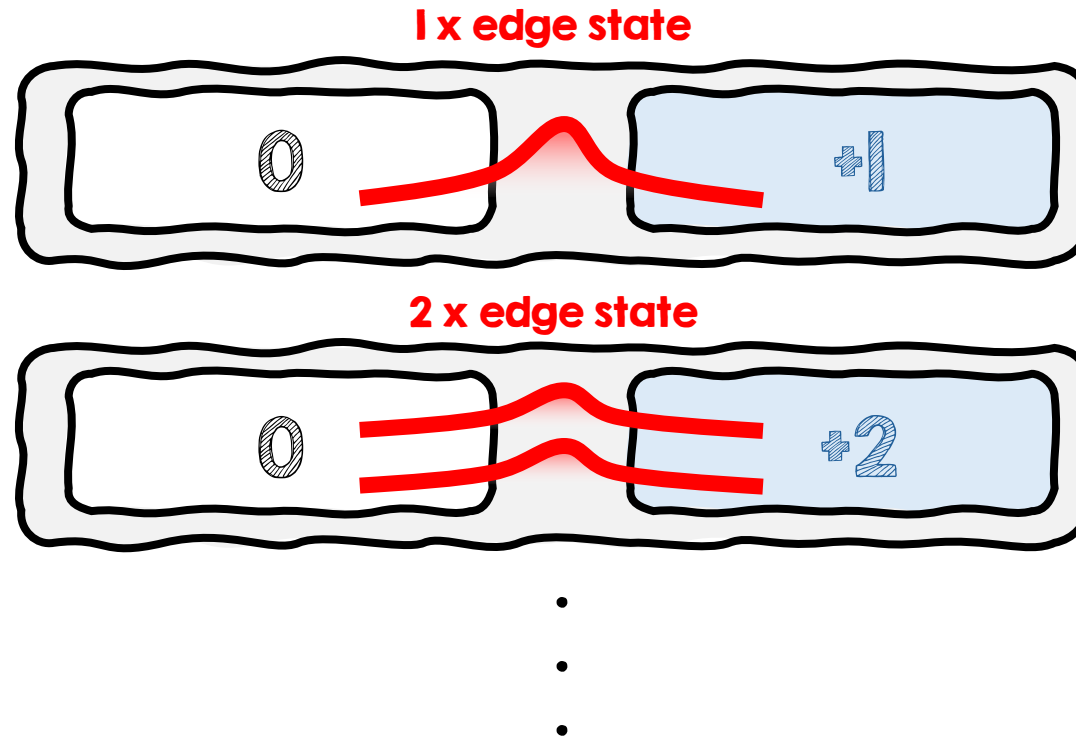
**Just like in your boiling kettle, water coexists with steam.
Topological phases also coexist in real space.**



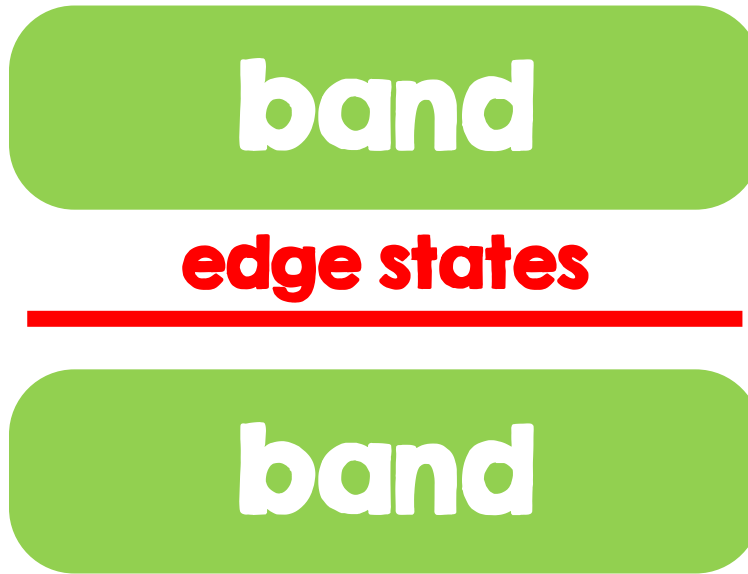
But unlike conventional water-steam system, topological phases carry charges to be vanished at the boundary.



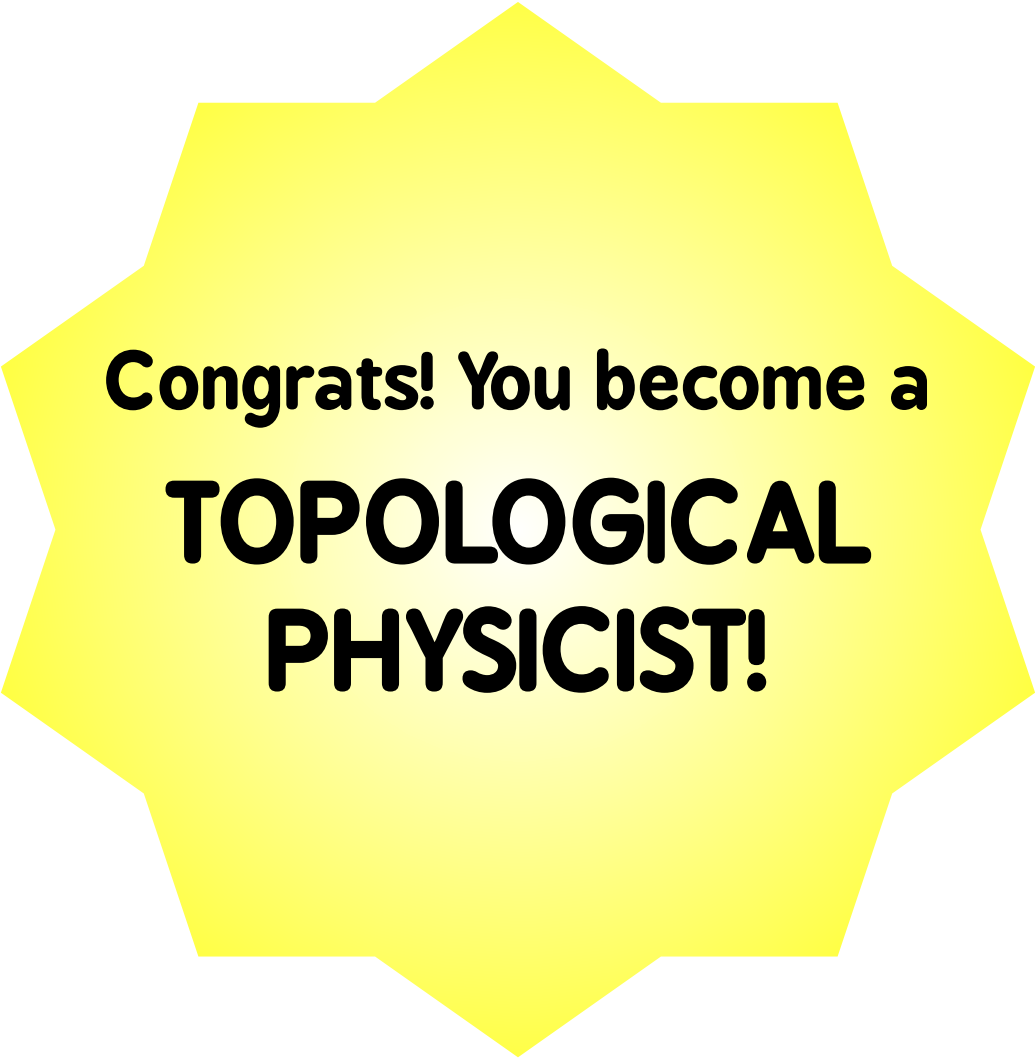
Something needs to stay at the boundary to cancel out the charge. This gives rise to “**edge states**”.



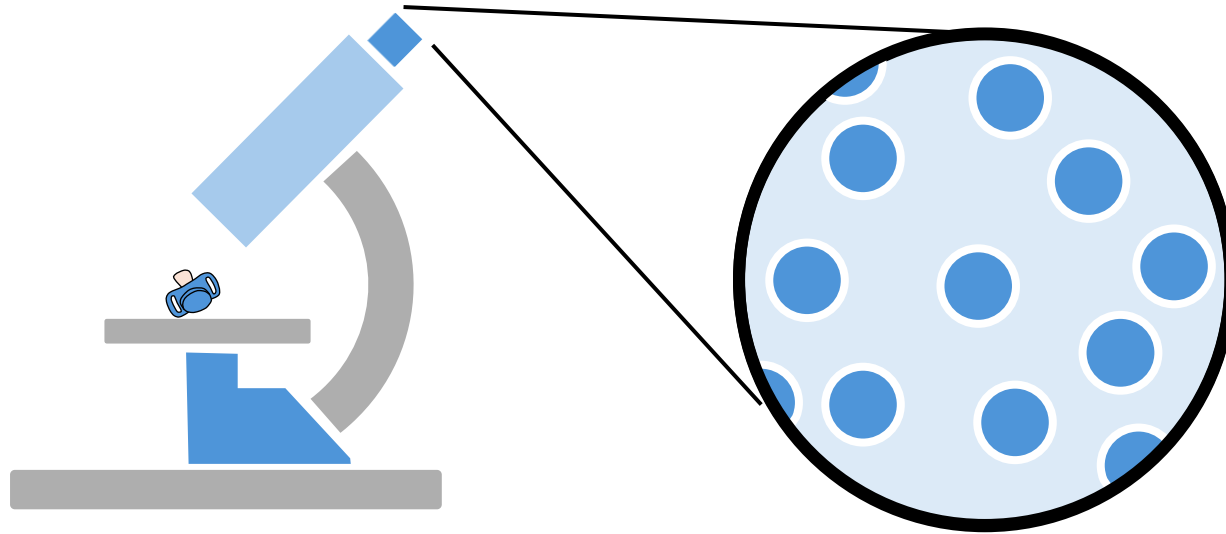
We know the number of edge states by bulk charges. This is called “**bulk-edge correspondence**”.



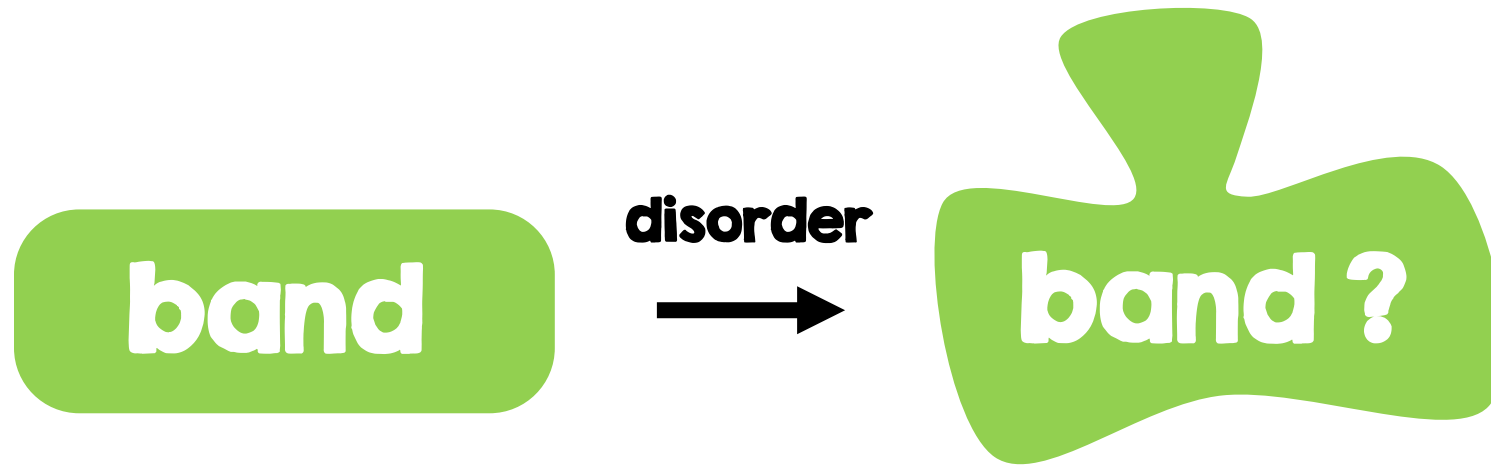
**As edge states reside where gap closing happens,
their energies usually appear in the gap.**



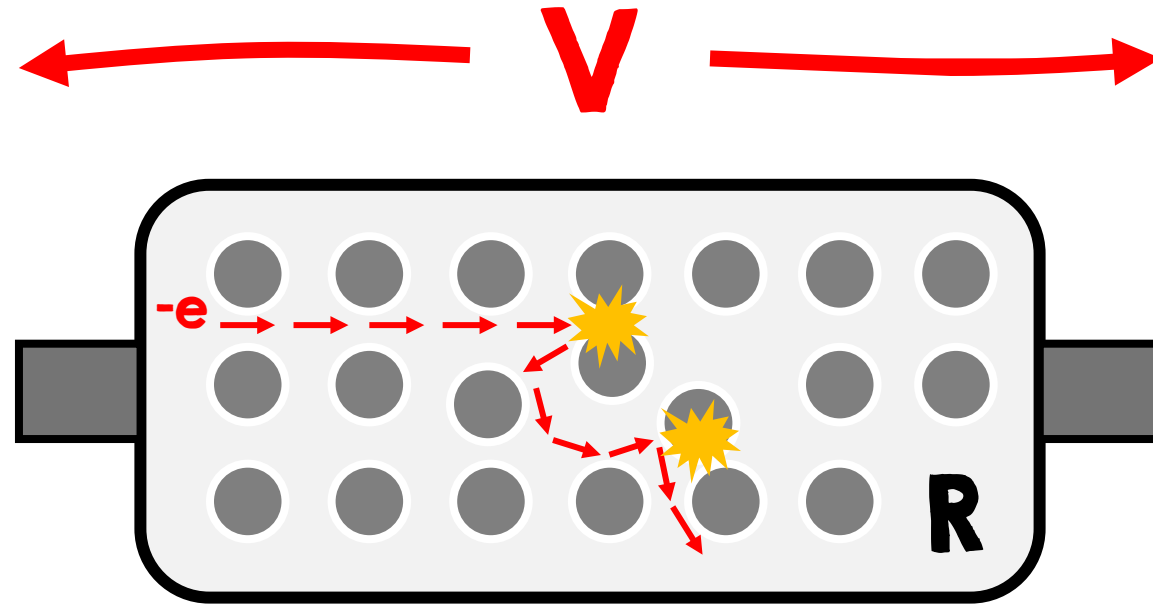
**Congrats! You become a
TOPOLOGICAL
PHYSICIST!**



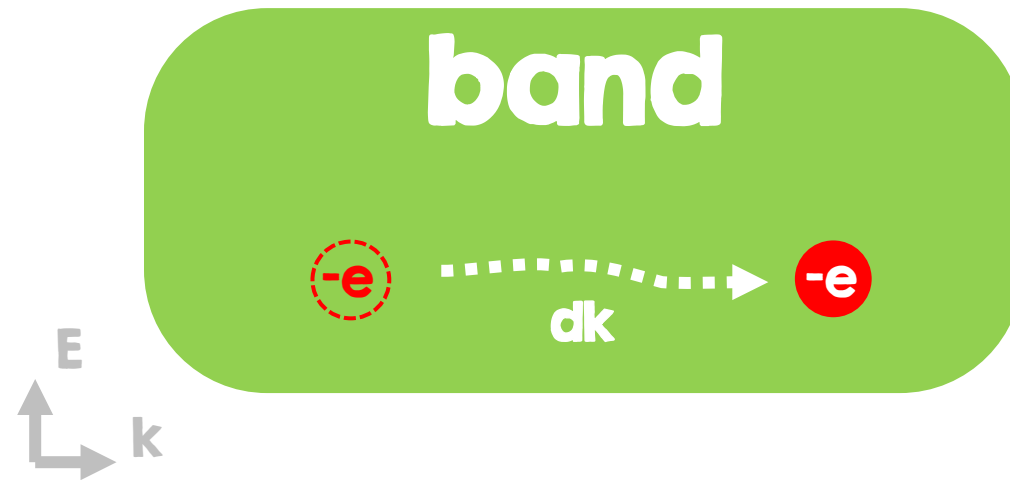
Disorder is everywhere in microscopic world.



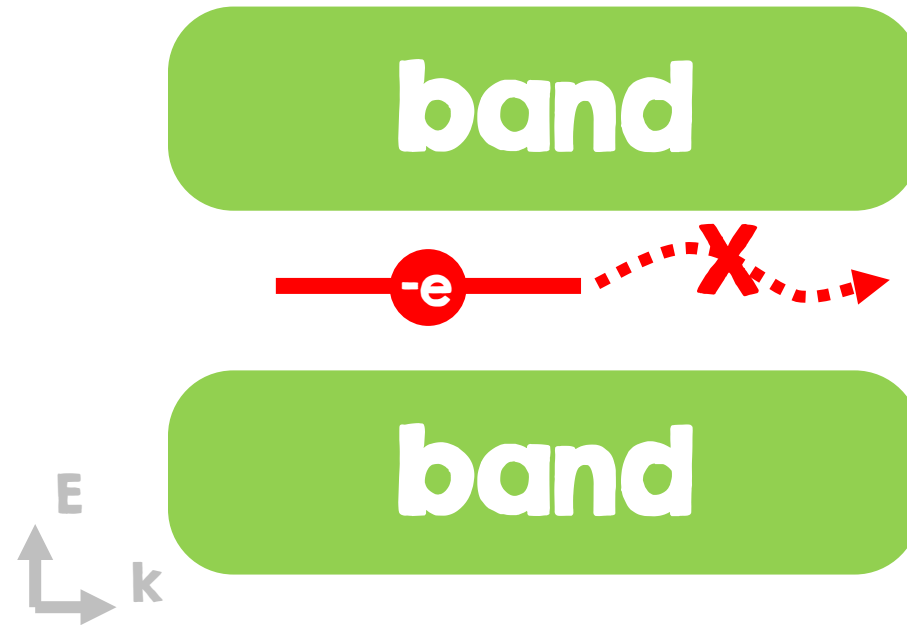
**Disorder in solids will squash the energy band.
This may largely change the solids' physical properties.**



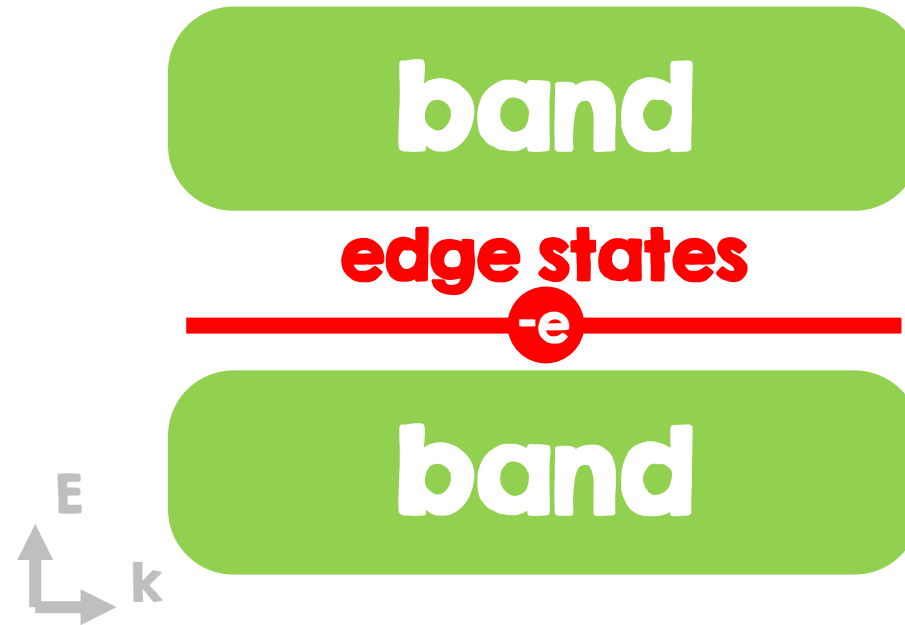
**This is one of the main contribution to metals' resistivity.
Electrons are kicked off the highway by defects.**



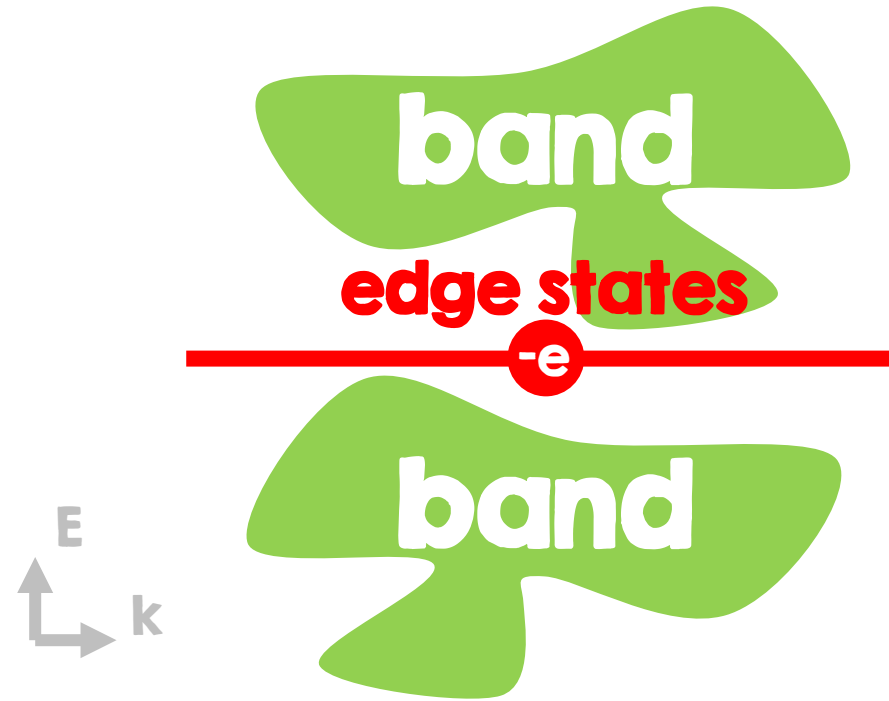
**From the band structure point of view,
electrons bounce into other degenerate states.**



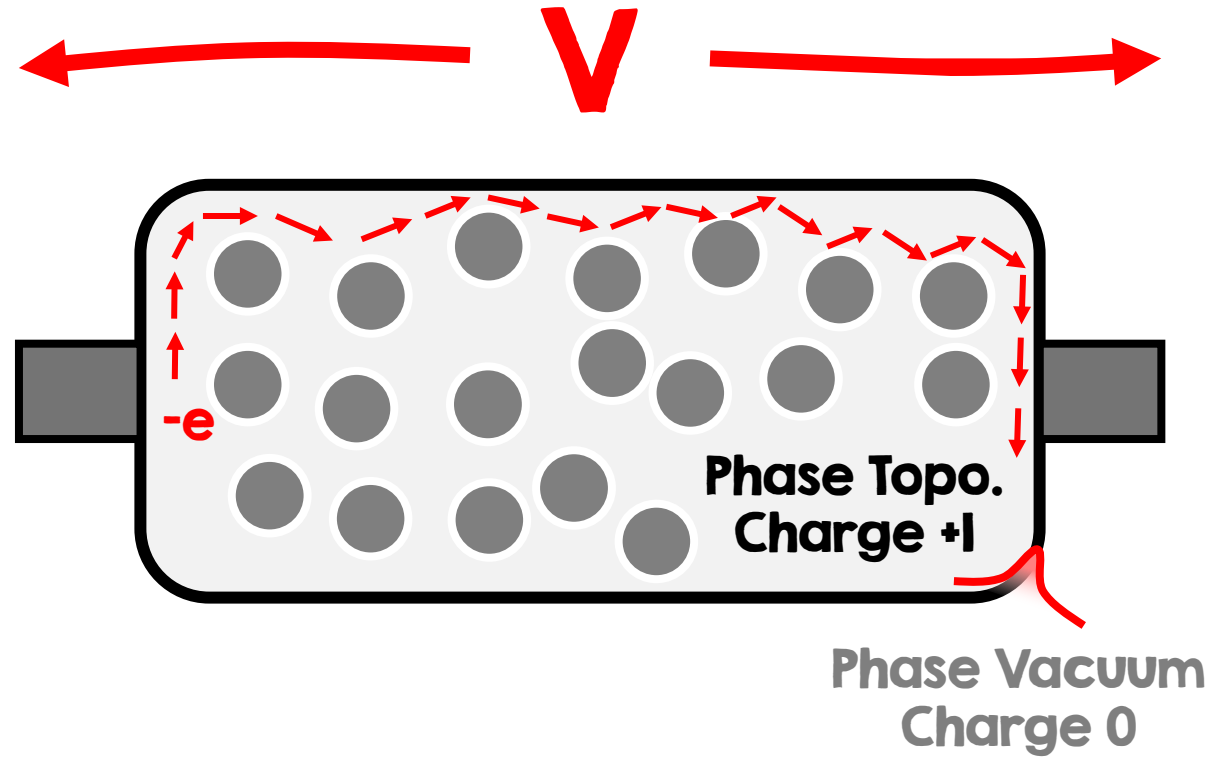
**One way to stop this transition is to eliminate the degeneracy.
For example, put one single **state** in the gap.**



Topological edge states are definitely one candidate.

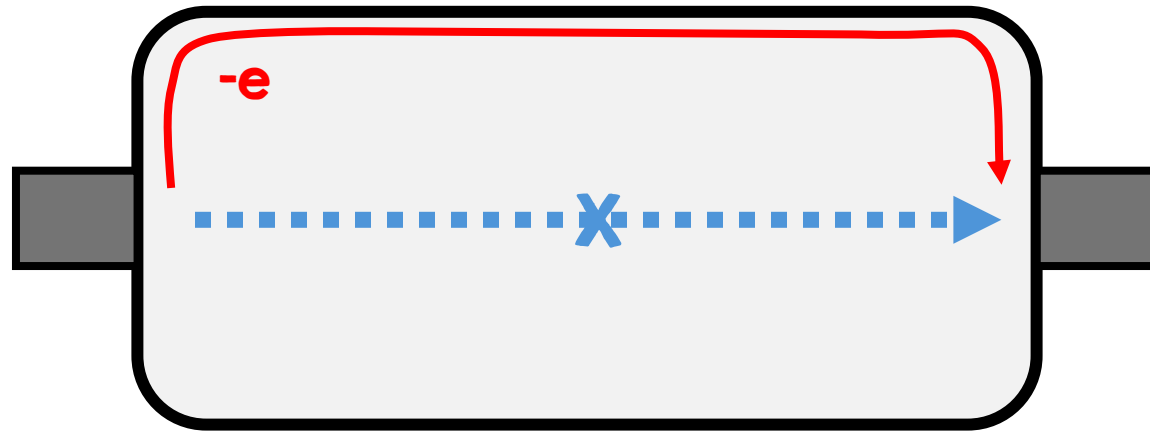


**Even if strong disorder squashes bands,
the topological structure may persist.**



**This time, electrons may wiggle on the edge,
but they never go backward.**

Topological Insulators



We call them **topological insulators**, as they usually insulate current in the bulk but superconduct on the edge.

A yellow starburst shape with eight points, centered on a white background. Inside the starburst, the text "Now you know" is written in a black, sans-serif font. Below it, the words "TOPOLOGICAL" and "INSULATORS!" are written in a larger, bold, black, sans-serif font, stacked on two lines.

Now you know
**TOPOLOGICAL
INSULATORS!**